



# NHH

INSTITUTT FOR SAMFUNNSØKONOMI

DEPARTMENT OF ECONOMICS

**SAM 16 2010**

**ISSN: 0804-6824**

JULY 2010

## Discussion paper

# The price of decentralisation

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# **The price of decentralisation**

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**Abstract:** This paper develops a model for analysing problems related to centralisation and decentralisation. The model is of the new economic geography type, in which there are agglomeration gains in cities but not in rural areas. These gains are counteracted by residential preferences. We show that, even though people have preferences for rural living, an unregulated market economy gives too little centralisation. This result holds even when city governments actively pursue policies to attract economic activities in order to make their city bigger. When allowing for cities of unequal size, a likely outcome is that big cities and rural areas will be overpopulated whereas smaller cities will be too few and too small.

**JEL classification:** R12, R13, H32, H41, R50

**Keywords:** Number of cities, size of cities, external economies, local public inputs, regional competition, agglomeration, welfare

# 1 Introduction

In almost all industrialised countries there has, as shown in table 1, been a strong, long-term trend towards urbanisation and increased centralisation. There are no indications that the trend is abating. In Norway, the share of the population living in urban areas has increased from 72% to 78% during the period from 1990 to 2008. Not only do people move to urban areas – the concentration of people *within* urban areas also increases. In Oslo e.g., the number of people per hectare increased from 37.9 to 42.3 from 2000 to 2009 (Næss et.al. (2009)). The rest of Europe is experiencing a similar development (although there medium-sized cities are growing at the expense not only of rural areas, but also of large cities).

	1800	1850	1910	1980
England	23	45	75	79
France	12	19	38	69
Germany	9	15	49	75
Europe	12	19	41	66
United States	5	14	42	65
Australia	-	8	42	80
Latin America	14	18	22	63
Third World	9	9	10	32

*Table 1: Urban share of population 1800-1980 (Crafts and Venables, 2003)*

Politically, the trend towards increased centralisation is seen as a problem in many countries. Norway is a case in point:

*The overall objective is to ensure equal living conditions throughout the country, maintain the settlement patterns and central features of the potential in all regions. People should have a real, independent choice regarding their place of residence. The Government facilitates economic development in all parts of the country and **encourages people to move to rural areas** (our italicising).*

(Norwegian Ministry of Local Government and Regional Development)

Therefore, to counter centralisation most governments protect or subsidise agriculture, undertake investments in infrastructure in scarcely populated areas, give subsidies or establish favourable tax regimes for firms located in rural areas, or relocate government agencies from the big cities.

An underlying assumption behind such policies is that there is too much centralisation<sup>1</sup>. There are, amongst others, *two* main economic arguments for this view. One is that there are diseconomies of scale in big cities due to e.g. pollution and congestion (see Kanemoto (1997) for a survey of the urban economics literature based on this perspective). The other, discussed e.g. by Martin (1999 a,b) and Puga (2002) within new economic geography models, is that there are negative pecuniary externalities when people move from rural to urban areas – those who leave do not take into account the negative effects on those left behind.

The literature on the new economic geography challenges the presumption that there is too much centralisation. While accepting that there could, at some point, be diseconomies of scale and negative pecuniary externalities, it argues that, under normal circumstances, these are more than offset by the real and pecuniary linkages which create positive external scale economies. The empirical evidence in this regard is strong, as centralisation has been a steady and universal trend for more than two hundred years. Had the negative external effects from centralisation been of the same order of magnitude as the positive scale economies, or had the pecuniary externalities in rural areas been of similar magnitude as the pecuniary externalities in cities, a spontaneous reversal of the centralisation trend would have been expected in at least some countries or over some extended time periods. Since neither are seen, there are good reasons to believe that the agglomeration gains in urban areas dominate any negative externalities in the cities and any positive externalities in rural areas.

If this is the case, the appropriate framework for discussing centralisation is one in which the driving force is the set of linkages which produce agglomeration gains of

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<sup>1</sup> We define centralisation as a “geographic centralisation of the population” in line with the definition used in the report “Sentraliseringens pris” (2009) written on request by the Ministry of Local Government and Regional Development.

the new economic geography type – see e.g. Fujita, Krugman and Venables (1999) or Ottaviano and Puga (1998) for excellent surveys of the new economic geography literature. During the last decade a large number of new economic geography articles have looked at problems related to decentralisation and centralisation policies (e.g. Matsuyama and Takahashi (1998), Baldwin et.al. (2005), Ulltveit-Moe (2007), Martin (1998, 1999a, 1999b), Martin and Rogers (1995), Puga (2001), Andersson and Forslid (2003), Forslid (2004)).

With real and pecuniary gains from agglomeration, the general presumption is that a market economy will give both too few and too small agglomerations (Norman and Venables (2004)) – in other words too few and too small cities. The purpose of this paper is to develop a framework for examining this presumption and to use it to see whether the presumption holds (a) when allowing for local city governments who actively pursue policies to attract more people in order to make the cities bigger, and (b) when allowing for equilibria with cities of unequal size.

The framework is one in which individuals have preferences for living either in the city or the countryside. Even though more and more people choose to move from rural to urban areas, many nevertheless express a genuine desire for rural living - their reasons being better recreational facilities, neighbourhood qualities, less pollution, less crowding etc. In economic models such non-economic considerations are usually ignored<sup>2</sup>, and job opportunities and wage differences are the only explanatory variables of workers' locational choices. A separate purpose of this paper, therefore, is to incorporate the fact that people value the place of living per se, to see whether it matters for the question of whether or not there is too much centralisation (which it turns out not to do).

In the cities, there are gains from agglomeration, so income levels there will be higher the larger the city. In rural areas, people work in agriculture; and their income levels depend on the total size of the agricultural sector (because the price of food falls with increased production). In the first part of the article, this framework is used to look at

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<sup>2</sup> An exception is Ludema and Wooton (2000), but their reason for assuming locational preferences is different. They assume locational preferences in order to ease the modeling of an upward-sloping labour demand curve.

an economy with one city in which the city government supplies tax-financed local services. One might think that the gains from agglomeration would make the city over-provide public goods in order to attract more people and that this could offset the standard result that the city is too small. That is not the case. At the margin, it is (by the envelope theorem) not possible to attract more people, so the only effect of expanding the public sector is to crowd out employment in the private sector, which is more costly when there are industrial agglomerations than when there are not. The presence of agglomeration gains will therefore actually make local governments supply *less* public services than they would otherwise do. The result that the city becomes too small and the rural sector too large is, if anything, strengthened.

In the second part of the paper, the model is extended to cover many regions and many cities. It is shown that the results regarding the number and size of cities are robust as long as people are mobile within regions only – in that case there will be at most one city per region; each will be too small; and some regions that ought to have cities will not have any. If we allow for interregional mobility as well (with individual preferences over regions as well as over rural vs. urban life), however, it could be that both big cities and rural areas will be overpopulated (relative to the optimum), while smaller cities could be both too few and too small.

## **2 An informal overview of the model**

We consider a closed economy consisting of rural areas and a number of cities. There are three sectors of production; the public, private and agricultural sectors. The public and private sectors are located in cities, the agricultural in rural areas (called the periphery).

There are a fixed number of inhabitants in the economy, each inhabitant supplying one unit of labour inelastically. Total labour supply thus equals the total number of inhabitants.

Workers are perfectly mobile within the economy, and choose location based on a consideration of where their total utility will be highest. Utility derives from the

consumption of private goods and from factors related to the place of residence itself. The utility derived solely from *living* in a specific place differ between individuals, but is exogenous to the model. We assume that people live in the same place as they work. Consumption goods are freely and costlessly traded within the economy. The consumption an individual enjoys therefore depends only upon the local wage rate.

Goods produced in the public sector are used as intermediates in the production processes of private firms. Examples of such goods might be infrastructure widely defined og general training experience. We call these goods public inputs. They are financed by local taxes levied on people living in the cities. Private firms produce a homogenous consumption good (which is used as numéraire) with labour and local public inputs as production factors. There are external economies of scale in the private sector, so productivity increases with the size of the sector (i.e. with the total volume of private sector production). The individual firm, however, does not take account of the scale economies because they are external to the firm. As a result workers will be paid the value of their *average* product which is lower than their *marginal* product. Hence there is a market failure in the labour market.

In the periphery agricultural production takes place with labour as the only factor of production.

In the first part of the paper (section 3) we study a single-city economy, i.e. an economy in which there is only *one* city in addition to the periphery. We describe the economy's production structure, and from this we derive labour demand. We continue by studying labour supply, which depends on relative wage rates and residential preferences. Having developed labour demand and supply, we study labour market equilibria and compare these to the efficient outcomes to see whether there will be too much or too little urbanisation. Finally, we find the optimal local supply of public inputs and discuss how any market bias regarding urbanisation is affected by local governments pursuing policies to supply the optimal amount of public inputs.

In the second part of the paper (section 4), we expand the analysis to a multi-region economy with an endogenous number of cities. The context is an economy consisting

of several regions, each of which is formally like the one studied in part one, and in which cities will be formed spontaneously so long as they are economically viable and stable. The purpose is to address the question of whether a free market economy produces too much or too little centralisation. In this context we also discuss possible effects of centrally initiated decentralisation policies.

### 3 A single-city economy

We consider an economy consisting of the periphery and *one* city. The economy has  $n$  inhabitants, each of whom inelastically supplies one unit of labour. The  $n$  inhabitants thus constitute total labour supply. Workers are perfectly mobile between the periphery and the city, and make a joint decision on where to live and work based on where their standard of living will be highest.

In this section we describe the production structure, employment and local public inputs supply in the single-city economy.

#### 3.1 Production and labour demand

##### 3.1.1 The private sector

In the private sector a large number of identical firms produce homogenous consumption goods with labour and a local public input as production factors. The aggregate production function is

$$(1) \quad x = \varphi(x, z)n^x,$$

where  $x$  is total production in the private sector,  $n^x$  total private sector employment,  $z$  the total amount of public inputs, and  $\varphi(x, z)$  is a function capturing labour productivity. The labour-productivity function captures the external economies and any interaction there might be between external scale economies and the supply of



local public inputs. Note that we do not model the sources of externalities explicitly; (1) should be interpreted as a reduced form of the market-linkage models developed in the “new economic geography” literature.

We assume that the labour-productivity function is increasing and concave in both arguments:

$$(2) \quad \varphi_x \equiv \frac{\partial \varphi(x, z)}{\partial x} > 0, \quad \varphi_{xx} \equiv \frac{\partial^2 \varphi(x, z)}{\partial x^2} < 0,$$

$$(3) \quad \varphi_z \equiv \frac{\partial \varphi(x, z)}{\partial z} > 0, \quad \varphi_{zz} \equiv \frac{\partial^2 \varphi(x, z)}{\partial z^2} < 0.$$

Solving (1) for the externality to express production as a function of labour and public inputs only, obtains

$$(4) \quad x = g(n^x, z).$$

To find the derivatives of this function (the social marginal products of labour and public inputs), we first differentiate equation (1),

$$(5) \quad dx = \varphi_x(x, z)n^x dx + \varphi_z(x, z)n^x dz + \varphi(x, z)dn^x,$$

which can be rewritten as

$$(6) \quad (1 - \varphi_x(x, z)n^x)dx = \varphi(x, z)dn^x + \varphi_z(x, z)n^x dz.$$

Solving this, gives

$$(7) \quad g_n \equiv \frac{\partial x}{\partial n^x} = \varphi(x, z) \left( \frac{1}{1 - \varphi_x(x, z)n^x} \right)$$

and

$$(8) \quad g_z \equiv \frac{\partial x}{\partial z} = \varphi_z(x, z) n^x \left( \frac{1}{1 - \varphi_x(x, z) n^x} \right),$$

respectively.

The terms in parentheses in equations (7) and (8) represent the external scale economies. With no economies of scale in the private sector, the marginal product of labour would have been  $\varphi$  and the marginal product of public inputs  $\varphi_z n^x$ . Due to external economies of scale, marginal products are higher.

The production function of firm  $i$  is

$$(9) \quad x^i = \varphi(x, z) n^i,$$

where  $x^i$  is firm  $i$ 's production and  $n^i$  is the number of workers employed by firm  $i$ . Because each firm is small relative to the entire industry, the effect of  $x^i$  on  $x$  is negligible, so the individual firm takes the function  $\varphi(x, z)$  to be constant, i.e. it perceives  $\varphi(x, z)$  as homogenous of degree zero.

The private consumption good is used as numéraire and the price is set equal to one. The inverse labour demand function from the private sector is given by the (firm) *perceived* value of the marginal product of labour:

$$(10) \quad w_D^c = \varphi(x, z).$$

Combining equations (4) and (10), private sector labour demand can be expressed as an indirect function of the wage rate and local public inputs supply,

$$(11) \quad w_D^c = \varphi(g(n^x, z), z).$$

The private sector labour demand curve is upward-sloping due to external economies:

Productivity and hence the wage rate (which equals the *firm perceived* value of the marginal product of labour) increases with the number of employees in the private sector. The slope of the labour demand curve is found by differentiating equation (11) with respect to  $n^x$ , giving

$$(12) \quad \frac{\partial w_D^c}{\partial n^x} = \varphi_x(x, z)g_n = \varphi_x(x, z)\varphi(x, z)\left[\frac{1}{1 - \varphi_x(x, z)n^x}\right] > 0.$$

Differentiating equation (11) once more shows that the demand curve is concave<sup>3</sup>:

$$(13) \quad \frac{\partial^2 w_D^c}{\partial (n^x)^2} = \varphi_{xx}g_n g_n + \varphi_x g_{nn} = \varphi_{xx}\left(\varphi\left[\frac{1}{1 - \varphi_x n^x}\right]\right)^2 + \varphi_x g_{nn} < 0.$$

Labour demand increases with the volume of public inputs supply. If the provision of public inputs increases, private firms increase production and therefore demand more labour. This is illustrated by the two demand curves in figure 1 (where  $\ddot{z}$  denotes a larger amount of public inputs production than  $\hat{z}$ ).

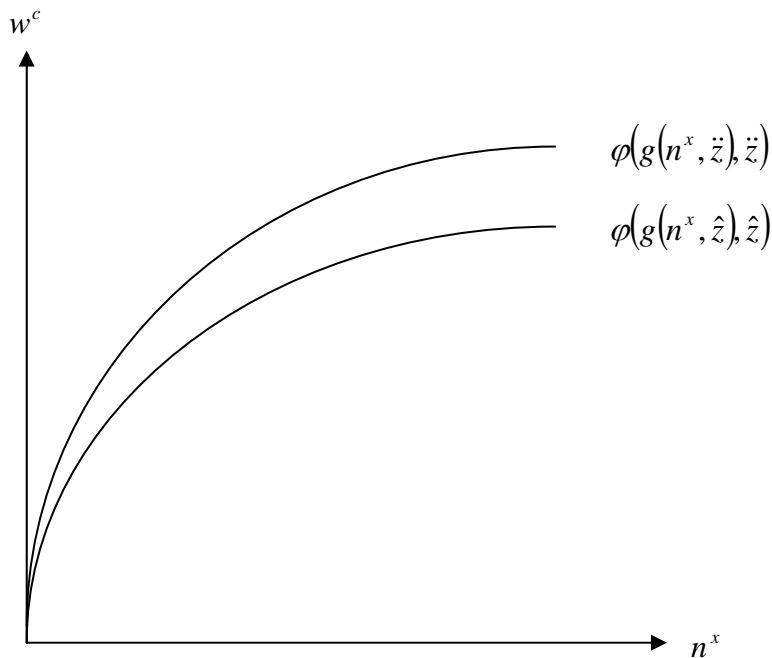


Figure 1: Private sector labour demand

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<sup>3</sup> Provided that  $g_{nn} < 0$ , i.e. that the production function is concave in  $n$ .

### 3.1.2 The public sector

Local public inputs are produced with labour as the only factor of production. Local governments decide on the amount of public inputs production. For the time being we do not consider how regional governments make their decisions, but simply take the chosen amount as given. Optimal public inputs supply is studied in chapter 3.4.

Assume constant coefficients in public inputs production. Public sector labour demand is  $n^z = az$ , where  $a$  is a positive constant. For simplicity we choose units such that  $a$  equals one, and then public sector labour demand becomes

$$(14) \quad n^z = z.$$

Total costs of public inputs production are

$$(15) \quad TC^z = w^c n^z,$$

where  $w^c$  is the wage rate in the city.

We assume that the city is self-financed (i.e. does not receive any grants from, or pay taxes to a central government). Public inputs production is financed through a uniform tax on the inhabitants of the city with per capita tax

$$(16) \quad t^c = \frac{TC^z}{n^c} = \frac{w^c n^z}{n^c} = w^c \left( \frac{n^z}{n^c} \right),$$

where  $n^c$  is the total number of workers in the city (observe from (16) that the tax decreases with the number of city inhabitants).

### 3.1.3 The agricultural sector

Agricultural production takes place with labour as the only factor of production according to the production function

$$(17) \quad y = y(n^y),$$

where  $y$  is total agricultural production and  $n^y$  agricultural sector employment.  $y(n^y)$  has constant returns to scale. We choose units so that the agricultural sector production function becomes

$$(18) \quad y = n^y.$$

Agricultural sector labour demand is implicitly given by the value of the marginal product of labour in agricultural production,

$$(19) \quad w_D^p = p^y y_n = p^y,$$

where  $w_D^p$  is the wage rate agricultural “firms” are willing to pay and  $p^y$  is the price of agricultural products.

We look at a closed economy and use the manufacturing output as numeraire. We also assume that demand for agricultural products is completely income-inelastic (derived e.g. from an additively separable utility function which is concave in the agricultural good and linear in the private good). The price of the agricultural good will then depend only (and negatively) on the quantity produced and sold, which in turn is determined solely by total agricultural employment. Thus, the value of the marginal product of labour in the agricultural sector, i.e. the agricultural wage, depends only, and negatively, on agricultural employment.

$$(20) \quad w^p(n^p); \quad w_n^p < 0$$

The agricultural sector labour demand curve is illustrated in figure 2.

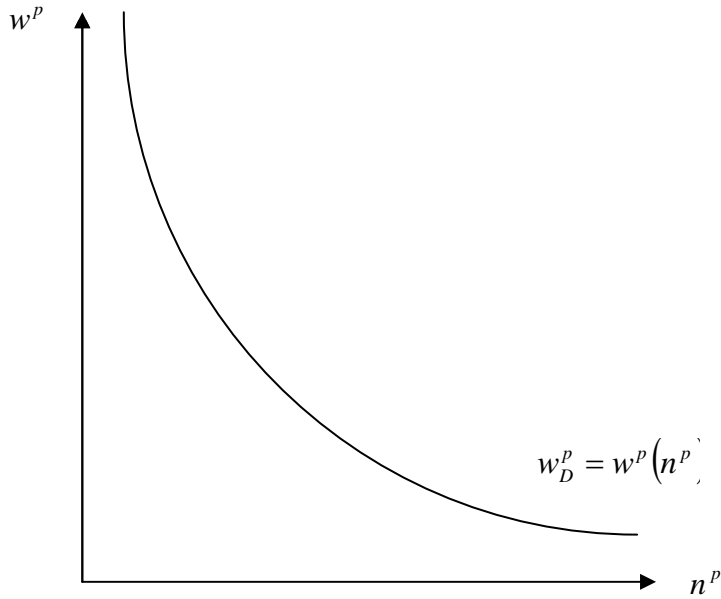


Figure 2: Agricultural sector labour demand

### 3.2 Residential preferences and labour supply

Workers are perfectly mobile between the city and the periphery, and choose to locate where their total utility will be highest. Utility derives from consumption of differentiated and agricultural goods, and from factors related to the place of residence. We assume that consumer goods (agricultural products and goods produced in the private sector in the city) are freely and costlessly traded within the economy, so consumers face the same prices wherever they live. The determinants of individual utility will then only be disposable income and place of residence.

Disposable income in the city is the wage minus the city tax

$$(21) \quad \omega^c = w^c - t^c = w^c \left( 1 - \frac{n^z}{n^c} \right).$$

while disposable income in the periphery is simply the wage

$$(22) \quad \omega^p = w^p,$$

where  $c$  denotes the city and  $p$  the periphery.

We do not model *why* people, cet. par., prefer to live in one place rather than another, but simply take this as exogenously given. Residential preferences are modelled by an individual-specific parameter  $\alpha_j$  which shows the additional consumer surplus person  $j$  gets from living in the periphery as compared to living in the city. The higher the value of  $\alpha_j$ , the higher is his preference for living in the periphery. Note that  $\alpha_j$  might be negative, in which case person  $j$ , cet.par., prefers to live in the city.

The utility functions of all individuals are assumed additive in consumption and place of residence, and the marginal utility of consumption constant (and equal to one). If person  $j$  lives in the city, his utility will therefore be

$$(23) \quad u_j^c = u(\omega^c) = \omega^c = w^c - t^c.$$

If he lives in the periphery his utility will be

$$(24) \quad u_j^p = u(\omega^p, \alpha_j) = \omega^p + \alpha_j = w^p + \alpha_j.$$

Any distribution of labour compatible with equilibrium must be such that the marginal inhabitant is indifferent between living in the city and in the periphery,

$$(25) \quad \omega^c = \omega^p + \alpha_M \Rightarrow w^c - t^c = w^p + \alpha_M,$$

where  $M$  denotes the marginal inhabitant.

Let  $F(\alpha_M)$  be the number of people who values the sheer pleasure of rural living higher than the marginal inhabitant does, i.e. the number of people for whom  $\alpha_j > \alpha_M$ .  $F(\alpha_M)$  is thus the equilibrium number of inhabitants in the periphery,

$$(26) \quad n^p = F(\alpha_M).$$

Those who do not live in the periphery, live in the city. Labour supply in the city is therefore

$$(27) \quad n^c = n - n^p = n - F(\alpha_M).$$

### 3.3 Equilibrium and efficiency

Labour market equilibrium obtains when (a) the marginal worker is indifferent between working in either sector and (b) the labour market clears, i.e. the sum of employment in the sectors add up to the total labour stock.

We perform a four-step analysis of labour market equilibrium, with the four steps being analyses when there are

1. No public inputs production and no residential preferences.
2. No public inputs production, but workers have residential preferences.
3. Public inputs production, but no residential preferences.
4. Public inputs production and workers have residential preferences.

Such step-wise analysis allows for isolation of different effects on the equilibrium conditions. The first case only serves as a benchmark, showing some of the well-known results from the new economic geography literature. Case 2 shows how residential preference affects existence, uniqueness and stability of geographic equilibria. Case 3 enables identification of the conditions under which there will be a local supply of public inputs and what effects that will have on productivity and urbanisation (eller centralisation?). The final case shows how public input supply and residential preferences can interact. It also sets up the complete model used (in section 3.4) to discuss optimal local policy.



### 3.3.1 No public inputs production, no residential preferences

We begin the equilibrium analysis by assuming that there is no production of public inputs and that workers have no residential preferences. This is the standard case discussed in the literature and can therefore serve as a point of reference.

No production of public inputs implies that there are two sectors of production: The private sector located in the city, and the agricultural sector located in the periphery.

No residential preferences and perfect mobility of workers between sectors imply that workers will enter the sector in which they get the highest income. In the agricultural sector workers are paid the value of their marginal product, as given by equation (20):

$$(20) \quad w^p(n^p); \quad w_n^p < 0$$

In the private sector workers are paid the value of their average product, as given by equation (11), but where  $z=0$ :

$$(11') \quad w^c = \phi(g(n^x)).$$

Workers have no residential preferences. Implicitly, therefore, they have no a priori preferences for working in either of the sectors (they are located at different places). Workers are perfectly mobile between sectors, and this mobility ensures that any labour market equilibrium is such that the wage rate is the same across sectors; i.e. the marginal product of labour in the agricultural sector equals the average product of labour in the private (agglomeration) sector,

$$(28) \quad w^c = w^p \Rightarrow \phi(g(n^x, 0), 0) = w^p.$$

The labour market must clear, which implies that the sum of employment in the two sectors must add up to the total labour stock,

$$(29) \quad n^c + n^p = n$$

The two conditions for labour market equilibrium when there is no public input production and workers have no residential preferences are given by equations (28) and (29).

Labour market equilibrium is illustrated in figures 3a and 3b.

In figure 3a we measure labour along the horizontal axis and wages/returns per worker along the vertical axis. The length of the horizontal axis is given by the total labour stock. From left we measure the number of workers in the private sector,  $n^c$ , from right the number of agricultural workers,  $n^p$  (which coincides with the number of inhabitants in the city and in the periphery, respectively). Private sector labour demand is given by the curve  $w^c(n^c)$  and agricultural labour demand by the curve  $w^p(n^p)$ .

As drawn, there are two equilibria satisfying equations (28) and (29), called B and D in figure 3a. These are equilibria because the return per worker is the same in both sectors *and* total employment in the two sectors add up to the total labour stock. Only D, however, is a stable equilibrium. The stability condition is that the slope of the agricultural sector labour demand curve is steeper than the slope of the private sector labour demand curve,

$$(30) \quad \left| \varphi_x(x, z) \varphi(x, z) \left[ \frac{1}{1 - \varphi_x(x, z) n_x} \right] \right| < |w^p|.$$

The likelihood of this happening increases with decreased agglomeration gains and the larger the returns to labour in agricultural production.

There is also the possibility that we end up in a situation in which the entire labour stock is employed in the agricultural sector - the point called A in figure 3a. In this situation, no worker will have any incentives to switch to the private sector because the wage rate in the agricultural sector is higher than what they may earn in the

private. As the two sectors are located at geographically different places this implies that everyone lives in the periphery.

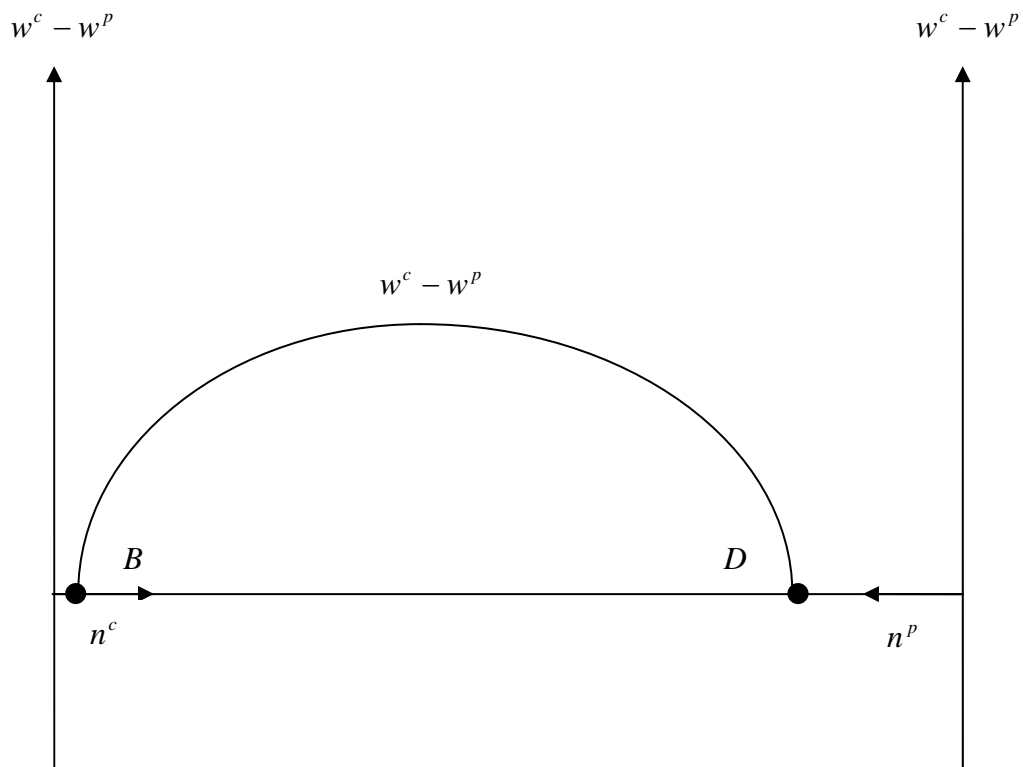
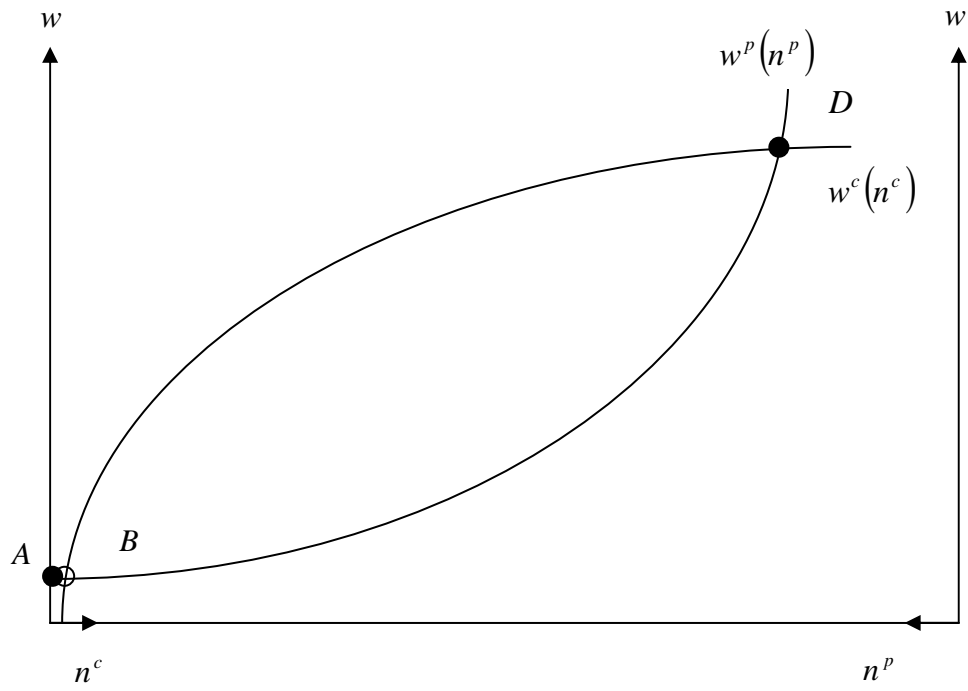
D is a diversified equilibrium, whereas the situation depicted by A is an equilibrium in agricultural production only. B is the critical mass of the city. If, for some reason or other, the size of the city is smaller than this, we end up in A. If the size is larger than this, we end up in the diversified equilibrium.

Labour market equilibrium may alternatively be illustrated as in figure 3b, which is derived from figure 3a. Figure 3b illustrates equilibrium by considering the wage differential between the private and the agricultural sectors. The wage differential,  $(w^c - w^p)$ , is given by the vertical distance between the two curves and in figure 3a. A, B and D in figure 3b correspond to A, B and D in figure 3a.

The stability condition, as given by equation (30), corresponds to a condition saying that the stable equilibrium is at the decreasing part of the wage differential curve,

$$(31) \quad \frac{d(w^c - w^p)}{dn^c} < 0$$

At the increasing part of the wage differential curve, the marginal economic gain from increasing the number of city dwellers is larger than the loss in residential surplus of the marginal inhabitant of the periphery. Hence, a stable equilibrium cannot occur at the increasing part of the wage differential curve.



Figures 3a and 3b: Labour market equilibrium, no public inputs production, no residential preferences

These results, the possibility of either a diversified or a concentrated equilibrium, are well-known from the literature on new economic geography - see e.g. Krugman (1991a) and Krugman (1991b). From this literature it is also well-known that equilibrium implies unexploited scale economies. This can be seen in figure 4. D corresponds to the diversified stable equilibrium D in figures 3a and 3b. We have drawn private and agricultural sectors labour demands, i.e. the firm perceived value of the marginal product of labour in private goods production and the value of the marginal product of labour in agricultural production, respectively. The *true* value of the marginal product of labour in private goods production is, however, larger than the individual firm perceives. The true value is given by equation (7),

$$(7) \quad g_n \equiv \frac{\partial x}{\partial n^x} = \varphi(x, z) \left( \frac{1}{1 - \varphi_x(x, z)n^x} \right)$$

and illustrated by the upper concave curve in figure 4. The efficient equilibrium is E, which implies higher private sector employment and lower agricultural, and hence that the scale economies are more fully exploited.

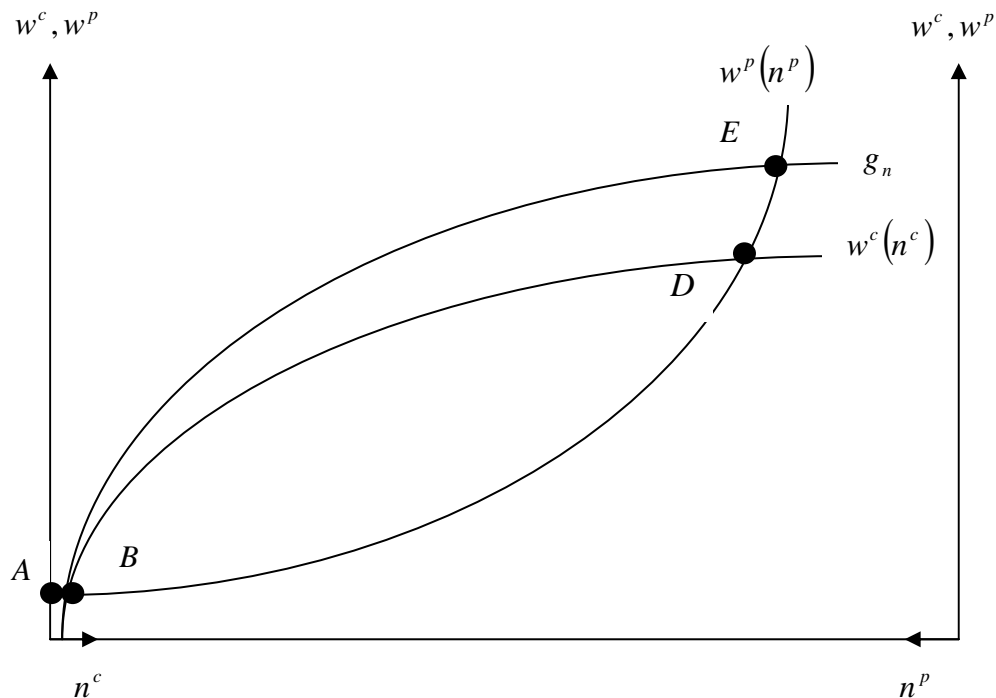


Figure 4: Unexploited scale economies in the stable equilibrium, no public inputs production, no residential preferences

### 3.3.2 Residential preferences, no public inputs production

When workers *do* have residential preferences, the wage rate alone no longer determines a worker's location. Residential preferences are represented by the individual-specific parameter  $\alpha_j$ . Workers are perfectly mobile and choose location based on where their total returns, including the one derived from place of living *per se*, will be highest. Mobility of workers ensures that the wage rate in the city equals the sum of the wage rate in the periphery and the marginal inhabitant's residential surplus derived from living in the periphery rather than in the city,

$$(32) \quad w^c = w^p + \alpha_M$$

Labour market clearing says that total employment in the private and agricultural sectors must add to the total labour stock,

$$(29) \quad n^c + n^p = n.$$

The two conditions for labour market equilibrium when there is no public input production but workers have residential preferences are given by equations (32) and (29).

Equilibrium is illustrated in figure 5. The  $\alpha$ -curve shows workers in ascending order with regards to preferences for living in the periphery.

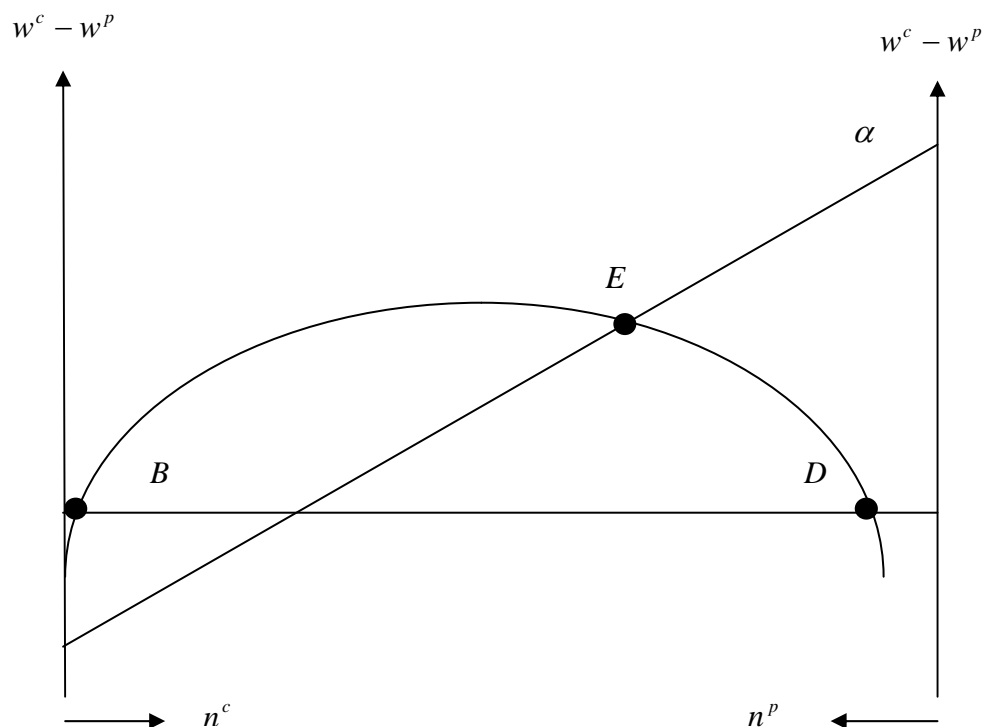
In figure 5 there is *one* unique equilibrium, E. To ensure a unique stable equilibrium, residential preferences must not be too weak. More precisely, the wage rate in the periphery must never be high enough compared to the wage rate in the city to induce the person who most highly values urban living to move to the periphery,

$$(33) \quad \alpha_1 < w^c(0) - w^p(n),$$

where 1 denotes the person who values living in the city highest.

If this condition is not fulfilled, there will be three possible equilibria, one unstable and two stable. The stable equilibria are the well-known ones – a concentrated equilibrium where everyone lives in the periphery and a diversified one with settlement in both the city and in the periphery.

This could be illustrated in figure 5, with a less steep residential preference-curve than the one depicted, alternatively a wage differential curve located further down.



*Figure 5: Labour market equilibrium with residential preferences: One unique diversified equilibrium*

We see that residential preferences affect equilibria in three different ways.

First, they make it less likely that there are multiple equilibria. As seen from figure 5, provided that residential preferences are not very weak, there will be *one* unique equilibrium, while there would have been three without residential preferences. For the same reason, the equilibrium allocation of people between the centre and periphery will be less sensitive to external shocks – residential preferences reduces

mobility and thus dampens the effects of shocks.

Second, with residential preferences, there will generally be an equilibrium wage gap between the centre and the periphery. If the marginal resident gets a higher “value added” from living in the periphery rather than in the city,  $\alpha_M > 0$ , the equilibrium wage rate in the periphery will be lower than in the city. If the marginal inhabitant values living in the city higher than in the periphery,  $\alpha_M < 0$ , the opposite happens – but empirically this is of little, if any, relevance. (Only if the marginal resident gets the same pleasure solely from living in the periphery and the city,  $\alpha_M = 0$ , will there be no wage gap between the two).

Third, the relative sizes of the city and the periphery differ from the sizes when workers have no residential preferences. If the marginal person gets a higher residential surplus solely from living in the periphery rather than in the city, then the equilibrium size of the city will be smaller than in the no-residential preference case. If, for the marginal inhabitant, residential surplus of living in the city is higher than the surplus of living in the periphery, then the opposite happens: The equilibrium size of the city will be smaller than in the no-residential preference case.

### **3.3.3 Public inputs production, no residential preferences**

The third case is one with public input supply, but in which workers do not have residential preferences.

Note first that public inputs production and supply have two opposing effects on the production by private firms: First, a “productivity effect”: Public inputs are used directly in the production processes of firms in the private industry, and the provision of public inputs therefore increases private productivity and thereby private firms’ labour demand. Second, it has a “crowding-out effect” in the labour market: Workers are required for public inputs production and so part of the labour stock will be publicly employed. The number of workers available to firms in the private sector decreases. This we call the “employment effect”.



To see these effects more clearly, recall first that a condition for labour market equilibrium is that the marginal inhabitant is indifferent as to where he lives and works,

$$(28) \quad w^c - t^c = w^p \Rightarrow \varphi(g(n^x, 0), 0) - t^c = w^p.$$

The wage rates must be equal in the two sectors. This is ensured by the mobility of workers.

Secondly, the clearing condition, that the whole labour stock is employed, must be ensured

$$(35) \quad n^z + n^x + n^p = n.$$

I.e. every person in the labour stock must be employed either in the public, the private or in the agricultural sector.

Labour market equilibrium for a given volume of public input production is illustrated in figure 6. With no public input production the diversified equilibrium is  $D_0$ . The employment effect of public inputs production is seen by a reduction in the size of the “bathtub diagram”. The length of the horizontal axis measured from the far right corner to the second right corner equals the number of public employees. From the left hand corner we measure the number of private sector workers, from the second right hand corner the number of agricultural sector workers. The isolated employment effect is seen by the horizontal shift, of length  $n^z$ , in the agricultural sector labour demand curve – the movement from equilibrium  $D_0$  to  $d$ .

The productivity effect of public inputs supply is seen by an upwards shift in the private sector labour demand curve, which changes the equilibrium from  $d$  to  $D_1$ .

The total effect of public inputs production and supply is the sum of the crowding-out and productivity effects. Without external scale economies, the two would pull in the same direction in terms of wages and employment – both would contribute to higher

wages in the city; and as a result, more people would move there. *With* external scale economies, however, the two pull in opposite directions. The direct productivity effect contributes to higher wages in the city. Increased public employment, however, does the opposite: By bidding people away from the private sector, it contributes to lower private sector productivity. This could completely offset the direct productivity effect, in which case the urban wage would fall and people would move out of the city.

To see the exact condition, suppose one person, initially employed in the private sector, is hired by the local government to increase production of public inputs. Losing one person reduces private sector productivity by  $\varphi_x g_n$ . In the public sector, the person produces an extra unit of  $z$ , which will raise private sector productivity by  $\varphi_z + \varphi_x g_z$ . For the city wage rate to rise, therefore, we must have  $\varphi_z > \varphi_x (g_n - g_z)$ .

It will, of course, never be a rational policy for the city government to produce public inputs (which would also require higher taxes) if the net effect is to lower the wage. If local public goods are supplied, therefore, we can be certain that the direct productivity effect dominates the crowding-out effect at the margin. We can go even further: The city will not increase the supply of public inputs unless the resulting increase in the wage level is at least as high as the necessary increase in the local tax. It follows that local public inputs, if provided, will have a positive effect on the size of the city, and a negative effect on the population in rural areas.

We shall discuss the optimal supply of local public inputs – including the question of whether local policy contributes to excess urbanisation – in greater detail in section 3.4

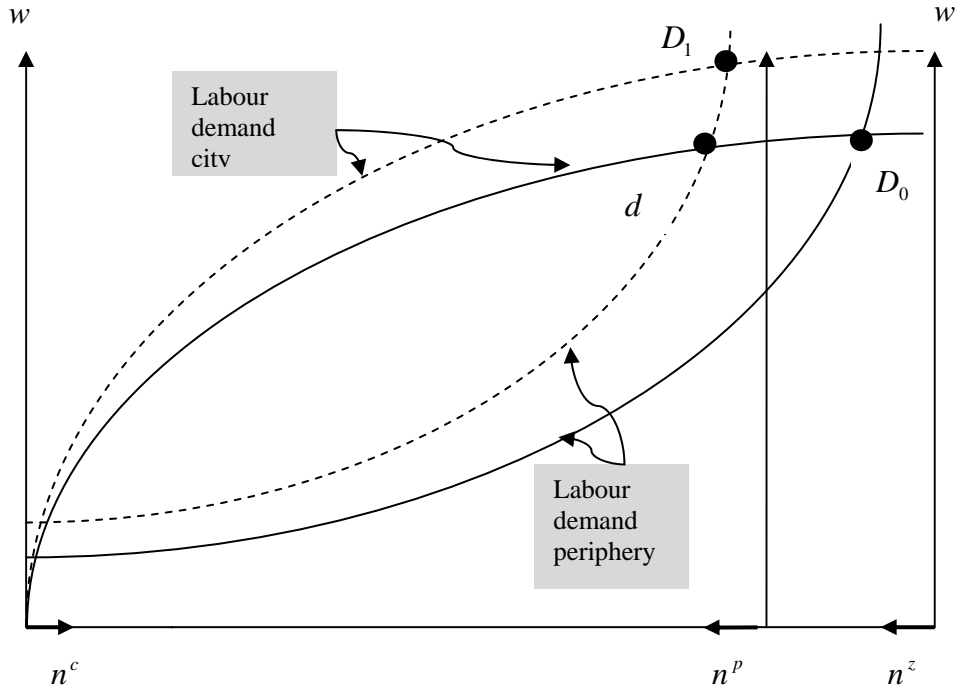


Figure 6: Labour market equilibrium, public inputs production, no residential preferences

### 3.3.4 Public inputs production and residential preferences

Let us finally, before we turn to the normative questions, set up the complete model with both public inputs production and workers with residential preferences.

Generally, labour market equilibrium obtains when the marginal worker is indifferent between working in either sector *and* the labour market clears.

The “indifference”-condition means that the marginal worker is indifferent between working in the public, private or agricultural sectors. The public and private sectors are located in the city whereas the agricultural sector is located in the periphery, and so the “indifference”-condition implies that the marginal worker is indifferent between living in the city or in the periphery. If he lives in the city his utility equals the private goods consumption he enjoys there (which, due to the assumptions that

everyone supplies one unit of labour inelastically and that the price of private goods are normalised to one, equals the net wage rate in the city),

$$(36) \quad u_M^c = w^r - t^c$$

If he lives in the periphery his utility is the sum of private goods consumption there (which equals the wage rate in the periphery) and the residential surplus derived from living in the periphery per se

$$(37) \quad u_M^p = w^p + \alpha_M .$$

The “indifference”-condition becomes,

$$(38) \quad u_M^p = u_M^c \Rightarrow w^p + \alpha_M = w^c - t^c$$

From (38) we find  $\alpha_M$ ,

$$(39) \quad \alpha_M = w^c - t^c - w^p$$

The second equilibrium condition is that the labour market clears, i.e. the sum of employment in the three sectors equals the total labour stock,

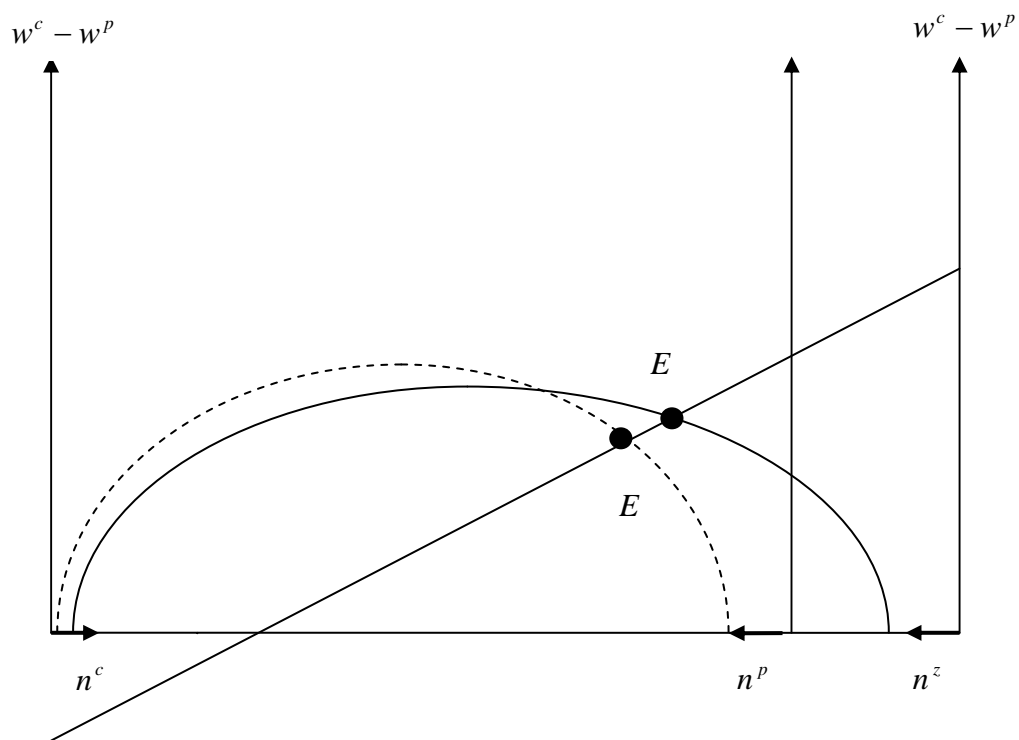
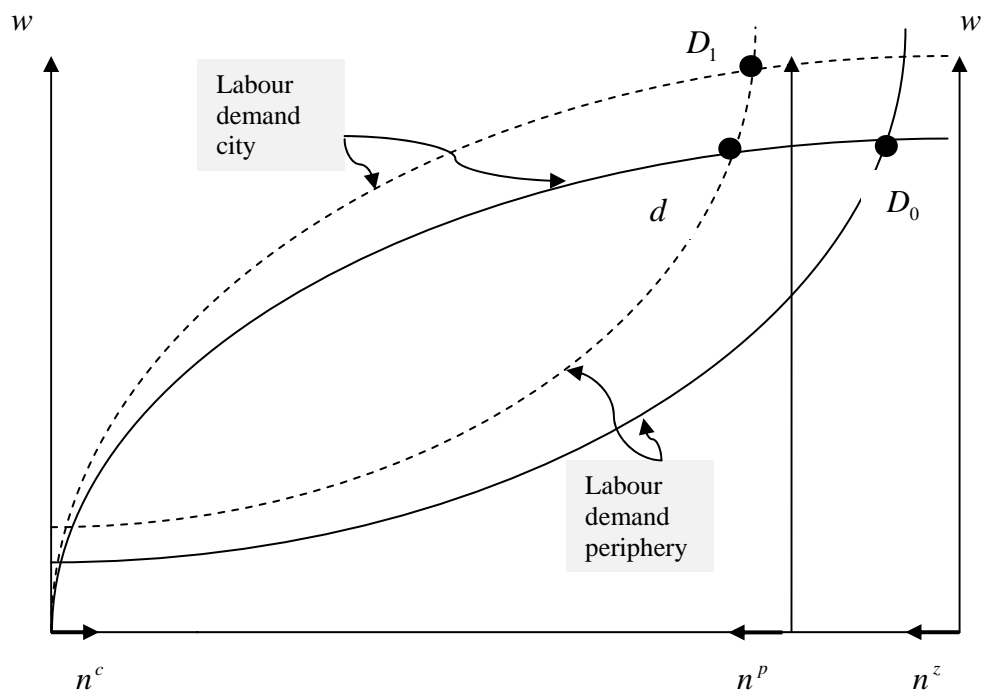
$$(35) \quad n^z + n^x + n^p = n .$$

When equations (39) and (35) hold; labour market equilibrium obtains. This is illustrated in figures 7a and 7b.

Figure 7a is a reproduction of figure 6. Figure 7b shows the wage difference between the city and the periphery, and is derived from figure 6. Without further assumptions, however, we cannot conclude as to whether the wage differential curve will shift downwards or upwards, i.e. whether the wage gap between the centre and the periphery will grow or decline. We know that employment in the agricultural sector

declines, causing a wage increase in the periphery. If the private sector employment also declines, then the wage rate in the city will fall and the wage difference clearly declines causing a downward movement of the curve. If, on the other hand, private sector employment increases then the wage gap may either increase or decrease. Labour market equilibrium is at the point where the residential preference curve crosses the wage differential curve – point E. Provided that residential preferences are not too weak, we get one unique stable equilibrium.

We see that there is one point in the figure that fulfils the two requirements for labour market equilibrium, point E, i.e. there is one unique equilibrium. Provided that the residential preferences are not too weak there will always be one unique equilibrium (see section 3.3.2).



Figures 7a and 7b: Labour market, public inputs production and residential preferences

### 3.4 Optimal public inputs supply

So far, we have taken the amount of public inputs supply as given, i.e. we have treated  $z$  as if it were exogenously given. The amount of local public inputs provision is, however, clearly a political issue. An important question in relation to the overall issue of centralisation is whether local governments will want to use public inputs supply in a way which attracts an excessive number of people to the city.

Local governments choose the amount of public inputs supply so as to maximise the welfare of their citizens. The utility of a representative resident in the city is given by his disposable income, as shown by equation (23),

$$(23) \quad u^c = w^c - t^c.$$

Inserting for per capita tax,  $t^c$ , from equation (16) and bearing in mind our choice of units such that public sector labour demand equals the amount of public inputs production,  $n^z = z$ , gives us the utility of a city dweller as

$$(40) \quad u^c = w^c \left( 1 - \frac{z}{n^c} \right).$$

We assume welfare is the sum of individual utilities. City welfare is thus

$$(41) \quad W^c = n^c \left[ w^c \left( 1 - \frac{z}{n^c} \right) \right].$$

Local governments choose the amount of public inputs production so as to maximise the welfare of its current inhabitants. As long as the number of inhabitants is given, this gives the same result as maximising the utility of a representative inhabitant, and so we may write the maximisation problem of local governments as

$$(42) \quad \max_z u^c = \max_z \left\{ w \left( 1 - \frac{z}{n^c} \right) \right\}.$$

The optimal amount of public inputs supply is such that there is no welfare gain from a marginal increase of public inputs supply:

$$(43) \quad \frac{du^c}{dz} = 0$$

which gives <sup>4</sup>

$$(44) \quad \frac{du^c}{dz} = -\frac{w^c}{n^c} + \left(1 - \frac{z}{n^c}\right) \left( \frac{\partial w^c}{\partial z} + \frac{\partial w^c}{\partial n^x} \frac{dn^x}{dz} \right) + \frac{\partial u^c}{\partial n^c} \frac{dn^c}{dz} = 0$$

Any optimum is such that the last term of equation (44) equals zero; i.e. there is no utility gain from a marginal increase in public inputs supply. No utility gain implies that there will be no migration either – no one gains from moving to or from the city. So, the optimal amount of public inputs supply is such that

$$(45) \quad \frac{dn^c}{dz} = 0.$$

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Inserting for  $\frac{dn^c}{dz}$  from equation (40) gives

$$\frac{du^c}{dz} = \frac{d \left( w^c \left\{ 1 - \frac{z}{n^c} \right\} \right)}{dz} = 0$$

Carrying out this differentiation gives

$$\frac{du^c}{dz} = -\frac{w^c}{n^c} + \left(1 - \frac{z}{n^c}\right) \left( \frac{\partial w^c}{\partial z} + \frac{\partial w^c}{\partial n^x} \frac{dn^x}{dz} \right) + \frac{\partial u^c}{\partial n^c} \frac{dn^c}{dz} = 0$$



By setting the last term of equation (44) equal to zero, the condition for optimal public inputs supply becomes

$$(46) \quad \frac{du^c}{dz} = -\frac{w^c}{n^c} + \left(1 - \frac{z}{n^c}\right) \left( \frac{\partial w^c}{\partial z} + \frac{\partial w^c}{\partial n^x} \frac{dn^x}{dz} \right) = 0$$

which gives the optimum condition<sup>5</sup>

$$(47) \quad \left( \frac{\partial w^c}{\partial z} - \frac{\partial w^c}{\partial n^x} \right) n^x = w^c,$$

i.e. optimal public inputs supply is such that the private sector marginal gain equals the direct costs of public inputs production. The left hand side of equation (47) is the private sector marginal gain from increased public inputs supply. Increased public inputs supply leads to increased private sector production. The value of this is  $w_z n^x$ . In order to produce the public inputs, however, some workers will have to be transferred from the private to the public sector. This reduces private production. The value of this is  $w_n n^x$ . Thus,  $w_z n^x - w_n n^x$  is the net value of a marginal increase in public inputs production. The right hand side is the direct cost of a marginal increase in public inputs production, namely the wage rate.

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$$\frac{du^c}{dz} = -\frac{w^c}{n^c} + \left(1 - \frac{z}{n^c}\right) \left( \frac{\partial w^c}{\partial z} + \frac{\partial w^c}{\partial n^x} (-1) \right) = 0$$

$$\frac{du^c}{dz} = -w^c + (n^c - z) \left( \frac{\partial w^c}{\partial z} - \frac{\partial w^c}{\partial n^x} \right) = 0$$

$$\left( \frac{\partial w^c}{\partial z} - \frac{\partial w^c}{\partial n^x} \right) n^x = w^c.$$

Rewriting equation (47) gives

$$(48) \quad w_z n^x = w + w_n n^x.$$

Equation (47) is a modified version of the Samuelson rule (Samuelson (1954)) where the left hand side is the sum of marginal values of public input and the right hand side is the social marginal costs of public inputs. The social marginal costs of public inputs equal the sum of the direct and indirect costs. The direct costs are the wage payments, the indirect costs are the private sector productivity costs (caused by reduced labour supply for private firms).

In the earlier discussion of the effects of local public inputs production, it was found that public inputs will not be produced unless they contribute to higher disposable income in the city, and that such production therefore will contribute to a larger city population, and a smaller rural population, than otherwise (see section 3.3.3). In that sense, local production of public inputs contributes to greater urbanisation.

That does not mean that local public inputs production (or, more generally, local public policy) contributes to excessive urbanisation, however. The potential for raising productivity through provision of local public inputs gives rise to a real economic gain which is in the interest of the economy as a whole to realise. To reap the benefits, more people must live in the city. Thus, the urbanisation effect of local public inputs provision does not reflect a market failure.

In fact, from equation (48) it follows that optimal local policies will not contribute to excessive centralisation. If policies are pursued to maximize per capita real income, they cannot, at the margin, have any effect on the size of the local population. The only way in which agglomeration affects optimum policies, therefore, is by raising the opportunity cost of resources used for public production (the second term on the right-hand side of (48)). It follows that the supply of public inputs, *ceteris paribus*, will be smaller, not larger, in the presence of private agglomeration effects.

## 4 A multi-region economy

We now extend the model to a multi-region economy, and assume there are  $R$  regions. Within each region there is a periphery.  $K$  regions also has a city, where  $K \leq R$ . The periphery and the cities are all formally like those studied in the previous sections. Specifically, there are external returns in private sector production. The number of cities is determined endogenously through a free-entry condition. The idea is that new cities will emerge if they are economically viable, so the equilibrium number of cities will be given by the equilibrium number compatible with a stable labour market equilibrium.

In the single-city economy labour market equilibrium was characterised by two conditions: Labour market clearing *and* the condition that the marginal inhabitant was indifferent as to where he lived and worked.

In a model with several cities, the same two conditions apply for any labour market equilibrium. In addition, there is the free-entry condition: The number of cities must be the maximum number consistent with equilibrium. Thus, labour market equilibrium in a multi-region economy is characterised by

- i. The labour market clears, i.e. the sum of employment in all cities and in the periphery (which equals the sum of private and agricultural sector employment, respectively) add up to the total stock
- ii. The marginal worker is indifferent between working in either sector, which is the same as being indifferent between living in either location
- iii. The number of cities is the maximum number consistent with the two preceding conditions

We begin by looking at these three conditions in some greater detail.

*i. Labour market clearing*

The first condition for labour market equilibrium is that the labour market clears in each region i.e. the sum of employment in the city (if any) and the periphery adds up to the total labour supply,

$$(49) \quad n = n_i^p + n_i^c, \quad i=1, \dots, R$$

where  $R$  is the number of regions,  $n$  the number of workers in each region,  $n^p$  the number of agricultural workers and  $n^c$  the number of city workers.

If there *is* a city in the region, labour supply in the periphery is given by the number of people who gets a higher residential surplus solely from living in the periphery than the marginal inhabitant does,

$$(50) \quad n_i^p = F(\alpha_i^M), \quad i=1, \dots, R$$

Inverting equation (50) gives the residential surplus of the marginal inhabitant as a function of the number of residents in the periphery,

$$(51) \quad \alpha_i^M = G(n_i^p), \quad i=1, \dots, R.$$

Inserting for  $n^p$  from equation (49) gives the residential surplus of the marginal inhabitant as a function of the number of city dwellers,

$$(52) \quad \alpha_M(n_i^c) = G(n - n_i^c), \quad i=1, \dots, R.$$

If there is *no* city in a region, all the inhabitants of the region work in the agricultural sector, so the number of agricultural workers equals the number of residents

$$(53) \quad n_i^p = n \quad i=K+1, \dots, R.$$

The total number of agricultural workers in the economy is the sum of agricultural

workers in the regions with and without a city, i.e.

$$(54) \quad n^p = (R - K)n + \sum_{i=1}^K n_i^p$$

where  $n^p$  is the total number of agricultural workers in the economy.

ii. “Indifference” condition

The second condition for labour market equilibrium is that the marginal inhabitant is indifferent between living in a city and in the periphery. This condition applies, naturally, only for those regions in which there *is* a city. The “indifference” condition is fulfilled when the utility of the marginal inhabitant is equal in the city and in the periphery i.e. when the wage rate in the city equals the sum of the wage rate and the residential surplus of the marginal inhabitant in the periphery,

$$(55) \quad w_i^c = w_i^p + \alpha_i^M \quad i=1,...,K$$

When this indifference condition is fulfilled, the residential surplus of the marginal inhabitant equals the wage gap between the city and the periphery,

$$(56) \quad \alpha_i^M = w_i^c - w_i^p$$

The city wage rate equals the value of the average product of labour in private production, as given by equation (11’), p.16

$$(11'') \quad w^c(n_i^c) = \varphi(g(n_i^c)).$$

Equation (11’’) is a modified version of (11’) where we have specified that we look at a single region  $i$ . The wage rate in the periphery is given by the value of the marginal product of labour in agricultural production,

$$(57) \quad w_i^p = p^y \left( \sum_{i=1}^R n_i^p \right).$$

Inserting for the number of inhabitants in the periphery,  $n^p$ , from equation (54), gives the wage rate in the periphery as a function of the number of city dwellers,

$$(58) \quad w_i^p = p^y \left( (R-K)n + \sum_{i=1}^K (n - n_i^c) \right) = p^y \left( Rn - \sum_{i=1}^K n_i^c \right).$$

### *Labour market clearing and “indifference” conditions*

Labour market equilibrium may now be summarised by

$$(59) \quad \alpha_i^M = w_i^c(n_i^c) - p^y \left( Rn - \sum_{i=1}^K n_i^c \right).$$

Equation (59) incorporates both the “clearing” and “indifference” conditions. The market clearing condition is fulfilled by saying that the number of residents in the periphery is given by the number of workers not employed in the cities (we have set  $n^p = n - Kn^c$ ) and the indifference condition is fulfilled by saying that the residential surplus of the marginal inhabitant is given by the wage gap between any city and the periphery.

Note from (59) that the city population (and thus also the agricultural population) must be the same in all regions with cities. In the following, therefore, let  $n^c$  and  $\alpha^M$  denote the common values for all regions with cities.

### *iii. Free-entry condition*

The third condition for equilibrium is the free-entry condition. Before considering it formally, consider first a different, but related question: How many cities are viable within any one region? As the inhabitants of a particular region would be indifferent between cities within the region (they have preferences for urban vs. rural life; they also have preferences for living in their particular region; but they have no preferences over different cities within the region), the answer is straightforward: Any equilibrium with more than one city would be unstable, in the sense that any

difference in size between the cities – however small – would make everyone move to the largest city. It is clear, therefore, that there can be at most one city in each region.<sup>6</sup>

The free-entry question, therefore, is the maximum number of cities – in different regions – which is consistent with equilibrium. Since the urban population must be the same in all regions with cities, the question is the maximum  $K$  consistent with equation (59), i.e. the maximum consistent with

$$(59') \quad \alpha^M = w^c(n^c) - p^y(Rn - Kn^c)$$

which says that the number of cities will be the maximum number consistent with equilibrium i.e.  $K$  is the maximum number such that (58') is fulfilled. This maximum number is the  $K$  for which

$$(60) \quad \frac{d\alpha^M}{dn^c} = \frac{d[w^c(n^c) - \{p^y(Rn - Kn^c)\}]}{dn^c}$$

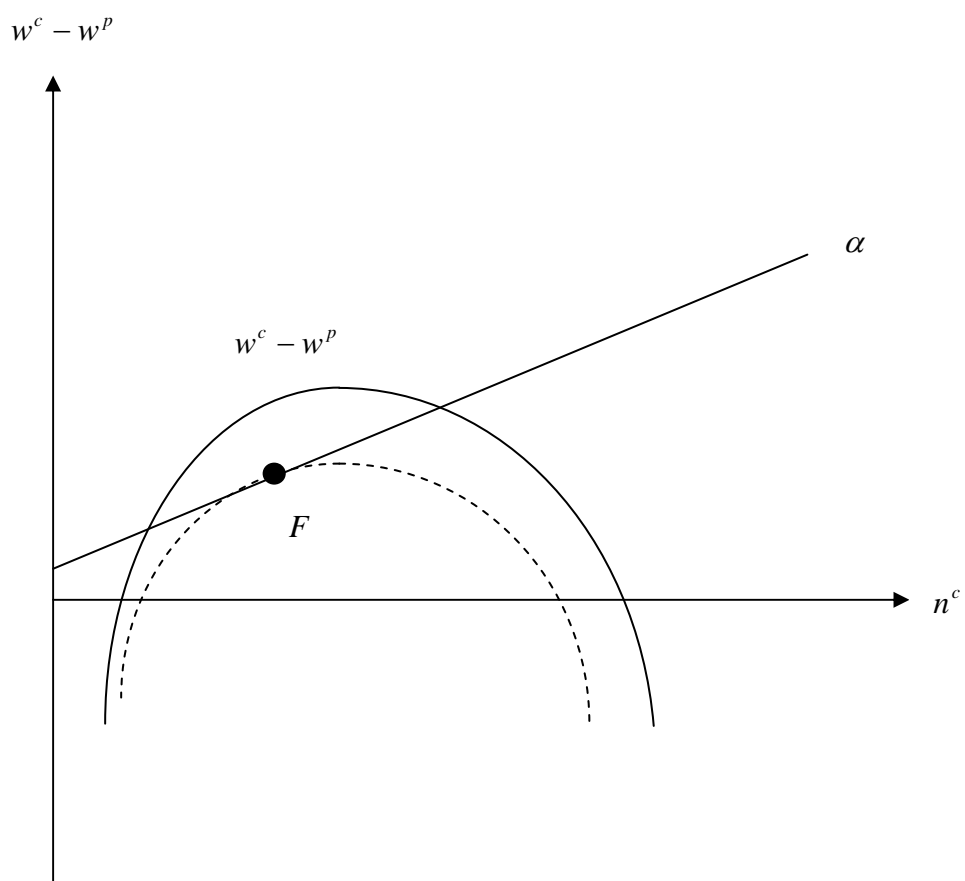
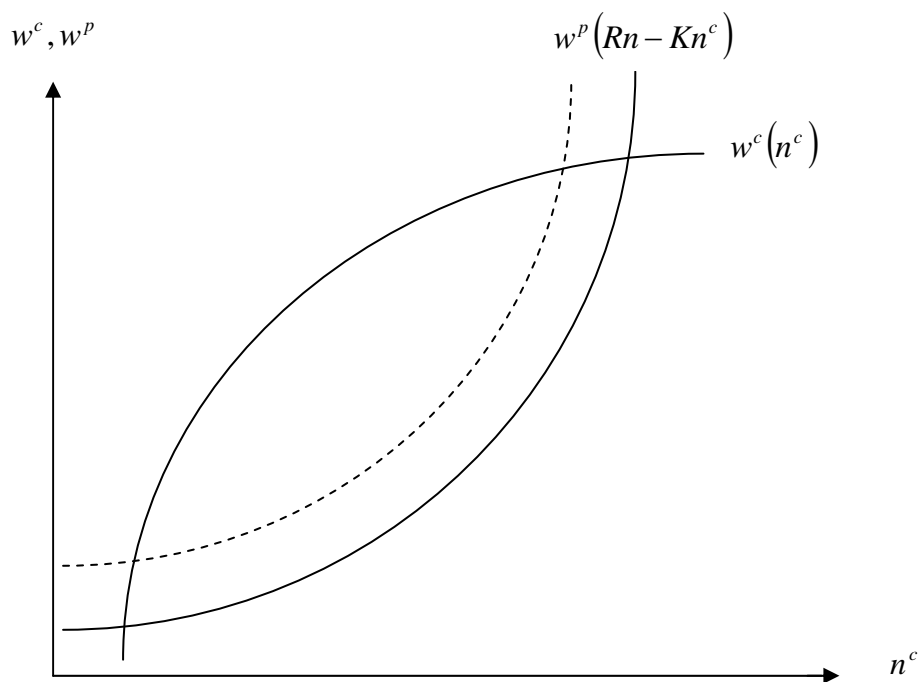
i.e. the equilibrium must be such that a marginal increase in the number of city dwellers (an external person) does not change the wage gap between any city and the periphery. Solving equation (60) we find

$$K = - \frac{\left( \frac{dw^c}{dn^c} \right)}{\left( \frac{p_n^y}{dn^p} + \frac{dG}{dn^p} \right)}$$

Labour market equilibrium in a region in which there *is* a city is illustrated in figures 8a and 8b.

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<sup>6</sup> I want to thank Kjetil Bjorvatn for pointing at this.



Figures 8a and 8b: Labour market equilibrium in a representative region with a city in a multi-city economy



In figure 8a we measure the number of city dwellers along the horizontal axis, and the wage rate along the vertical axis. Labour demand in the city is shown by  $w^c(n^c)$ , and labour demand in the periphery by  $w^p(Rn - Kn^c)$ . Labour demand in the periphery is decreasing in the number of workers in the periphery, and hence increasing in the number of city workers (the more people live in the city, the fewer live in the periphery).

Increasing the number of cities causes an upward shift in the labour demand curve in the periphery, as illustrated by the movement of  $w^p(Rn - Kn^c)$  from the solid to the dotted curve. In order to understand the mechanism behind this shift, the following thought experiment might be useful: Imagine that a new city is established in a region in which there is no city in the first place. Some inhabitants (of this region) will move to the new city. Holding the number of city dwellers in all other regions constant, the total number of agricultural workers decreases as a result of the new city establishment. As a result, total agricultural production declines causing an increase in the price of agricultural products. This price increase induces an increase in the value of the marginal product of labour in agricultural production, as reflected in an upward shift of the agricultural labour demand curve in any region.

Figure 8b is derived from figure 8a, and hence also applies to a representative region in which there *is* a city. We measure the number of workers in the city along the horizontal axis and the wage differential between the city and the periphery along the vertical axis. Labour market equilibrium is a.o. characterised by the “indifference condition”; i.e. the marginal inhabitant is indifferent between living in the periphery or in the city. The “indifference condition” is fulfilled when the residential surplus of the marginal inhabitant equals the wage differential between the city and the periphery. In figure 8b, at a point in which the two curves cross. An increase in the number of cities leads to higher prices of agricultural products, inducing a wage increase in the agricultural sector. The wage differential between the city and the periphery thus decreases with the number of cities, causing a downward movement of

the wage differential curve – as illustrated by the movement of the wage differential curve from the solid to the dotted curve in figure 8b. New cities will emerge until the point in which the wage differential and the residential preference curves are tangent. The maximum number of cities compatible with equilibrium is thus the one in which the wage differential curve and the residential preference curve are tangent – point F in figure 8b.

Several features of equilibrium are worth noting:

First, there will typically be cities (all of equal size) in some, but not all, regions. There will also be wage (and income) difference between regions in which there *is* a city and regions in which there is no city. This can be seen most clearly from figure 8a. The wage rates in regions in which there is a city will be  $w^c$  and  $w^p$  in the city and in the periphery, respectively. In regions where there is no city the wage rate will be  $w^{no\ city}$ .

Second, there is very little a central government can do to eliminate these income differences. Since the number of cities is the maximum number compatible with equilibrium, attempts at establishing urban agglomerations in more regions will at best mean that an existing city in another region is no longer viable. Similarly, if a central government attempts to eliminate the income differences through subsidies to agriculture, the only effect will be to reduce the equilibrium number of cities – but the income differences between regions with and without cities will remain.

Third, the equilibrium implies that the scale economies are not fully utilised. Equilibrium is at a point in which the wage differential curve is rising. This means that if the volume of private sector production were to increase ( $n^c$  increases) then the wage rate in the cities would increase due to higher productivity. In other words, the equilibrium implies that the cities are too small, and thus confirms the new-economic-geography presumption that there is too little, not too much, urbanisation.

## 5 Extension: Interregional mobility

The multi-region model in the previous section has a very rigid structure in that people are assumed to be perfectly mobile within regions, but not between them. As an extension, we shall in this section consider a model with both interregional and intraregional mobility. We assume that people have residential preferences along two dimensions. First, they have regional preferences, which we model as a preference for living in a particular region (but indifference between all the others). Second, people have preferences about rural vs. urban living. These rural vs. urban preferences are modelled in exactly the same way as residential preferences were modelled in the previous sections. We do not develop the model fully, but sketch it in sufficient detail to see possible outcomes and discuss economic policy.

Let  $\beta$  denote a person's preference (i.e. willingness to pay) for living in the region they come from. We assume that everyone is indifferent about all other regions than the one they come from. So, each person has a preference  $\beta$  for living in the region they come from in addition to the preference  $\alpha$  for urban vs. rural living. The utility functions are

$$(61) \quad U = u(y) + x + \alpha + \beta.$$

Let  $H(\alpha, \beta)$  be the cumulative density function over rural and regional preferences. I.e.  $H(\alpha, \beta)$  is the number of persons in a region with a rural preference parameter equal to or larger than  $\alpha$  and a regional preference parameter equal to or larger than  $\beta$ .

Workers are *perfectly* mobile both within and between regions. There will be a number of conceivable equilibria: A perfectly symmetric equilibrium with cities in all regions and no inter-regional movements, equilibria where some regions do not exist at all, equilibria with complete specialisation (complete urbanisation in some regions and agricultural production only in others), etc.

Our focus is on policy rather than on the pure theory of spatial equilibrium.

Therefore, we do not make an attempt to trace all of these equilibria, but instead focus on the ones we think are empirically most likely: An equilibrium with cities in some (but not all) regions, with workers moving from purely rural regions and with an asymmetry between the urban regions which results in one large and a number of smaller cities.

Why is this equilibrium possible and likely? An equilibrium with no inter-regional mobility implies that there are cities in some, but not in all, regions. People living in the periphery in a region and who have a low regional preference (low  $\beta$ ) and a low preference for living in the periphery (low  $\alpha$ ) will have a lower utility than they would get if they could move to a city in another region. If mobility is possible, therefore, they will move.

Where will they move? They are indifferent to other regions, so they will move to the city which offers the highest wage. Initially, i.e. with no mobility, all cities offer the same wage rate and we may therefore expect the migrants to be evenly distributed between cities. An even distribution is, however, not stable. If, for some reason, one city gets one more immigrant than the others, the wage rate would become slightly higher there than elsewhere. This would make all the migrants leave the other cities in favour of the higher-wage city.

Migrants from purely rural regions (regions with no city) will therefore move to *one* city only. Other cities will, however, continue to exist (inhabited by people with preferences for city-life *and* strong regional preferences), but they will have no inhabitants from other regions. Compared to the situation when people do not move between regions, the other cities will be smaller because some people (those with low regional preferences) will move from the smaller to the large city.

A likely outcome, therefore, is a rural-urban hierarchy where some regions are purely rural, some have small cities and one region has a large city. This is the outcome that will be our focus and the one we use as the basis for policy analysis. In order to analyse policy we need to formalise this equilibrium in more detail. Let subscript  $A$  denote the purely rural/agricultural regions,  $S$  the regions with small cities and  $L$  the

region with a large city.

Consider first the purely agricultural regions. A person from such a region will, if he stays in the region, get utility

$$(62) \quad U_A = p^y + \alpha + \beta .$$

If this person moves to the large city, his utility will be  $w_L$ . The marginal resident of a purely agricultural region will be given by

$$(63) \quad (\alpha + \beta)_A^M = w_L - p^y .$$

The number of inhabitants in each of the purely rural regions will be the number of people for which  $(\alpha + \beta) > (\alpha + \beta)_A^M$ .

Next, let us consider the regions with small cities. Some people in these regions will choose to live in rural areas and work in the agricultural sector. They get utility

$$(64) \quad U_S^y = p^y + \alpha + \beta .$$

Some will choose to live in the small city. Their utility will be

$$(65) \quad U_S^x = w_S + \beta .$$

The last group are those who prefer to move to the large city, which gives them utility  $w_L$ .

Thus, there are two marginal person(s) in the regions with a small city: Those who are indifferent between living in the periphery or the city within the region, *and* those who are indifferent between staying in the region or moving to the large city. We assume that the preferences are such that only the “city-lovers” are potential migrants to the large city i.e. we assume that there is a sufficiently strong correlation between

rural and regional preferences to prevent migration from the agricultural sector in small-city regions to the large city.

With this assumption, the marginal agricultural worker is the one with rural preferences,  $\alpha_s^M$ , equal to

$$(66) \quad \alpha_s^M = w_s - p^y$$

and the marginal migrant from the small-city regions is the one with regional preferences,  $\beta_s^M$ , given by

$$(67) \quad \beta_s^M = w_L - w_s.$$

The number of agricultural workers in each of these regions is the number of people for whom  $\alpha > \alpha_s^M$ . The number of emigrants is the number of people for whom  $\beta < \beta_s^M$ . The rest will be city residents.

Finally, let us consider the region with a large city. The population in this region consists of three groups: First, local people with a low rural preference; second, immigrants from small-city regions with a low rural preference. And third, immigrants from purely rural regions with weak regional or rural preferences.

The rural population are those people for whom  $\alpha > \alpha_L^M$ , where

$$(68) \quad \alpha_L^M = w_L - p^y.$$

The city population is the rest of the local population plus those from purely rural regions with  $(\alpha + \beta) < (\alpha + \beta)_A^M$  plus those from small-city regions with  $\beta < \beta_s^M$ .

The exact expressions for the number of people are not possible to asses without further assumptions regarding  $H(\alpha, \beta)$ . For our analysis, however, we do not need to

know the exact number.

Consider in this setting the effects of regional policy. Suppose that central governments want to promote decentralisation i.e. they want to encourage rural preference-people to move to rural areas, and do so by subsidising agriculture. The purpose of this chapter is to briefly describe how such a policy may be analysed within the theoretical framework developed in this paper.

Subsidising agricultural production leads to higher profitability in the agricultural sector, and an upwards shift in the inverse demand function for labour in the agricultural sector. Migration from purely rural regions to the large city will definitely be reduced. As a result of the reduced immigration from rural regions, the large-city wage rate declines.

The agricultural subsidy also leads to higher agricultural employment in the regions with a small city, which might lead to a reduction in the number of viable small cities. If this happens, migration to the large city from small-city regions could increase. (Some of those originally living in small cities no longer have any city in their preferred region. If they love living in cities they probably choose to move to the large city instead of into the periphery in their “home-region”.)

This last effect, increased immigration to the large city from previously small-city regions, might be so large that the net outcome is both increased agricultural employment and a bigger large city. The losers would be the small-city regions.

## 6 Conclusions

In this paper we have developed a model for analysing centralisation and decentralisation policies. The model is of the new economic geography type, in which there are gains from agglomeration in cities - acting as a centrifugal force. These agglomeration gains are counteracted by residential preferences - acting as a centripetal force. In the basic model, we establish the equilibrium conditions. We find that - in contrast to the well-established new economic geography models - there will be one unique stable equilibrium provided that residential preferences are not too weak. Generally, there will also be a wage gap between the city and the periphery.

Having developed the formal model, we first use this framework for studying the policies of a city government providing a locally tax-financed public input. The aim of the policy is to attract economic activity to the city. We find that the presence of agglomeration gains makes the government *undersupply* local public inputs. The reason is that public inputs production - due to the agglomeration effects - *raises* the opportunity costs of resources used for public production. This contrasts with previous results from new economic geography models studying public policies. In this literature findings show that there are reasons to believe in a “race to the top” regarding local taxes (and hence the supply of publicly provided goods and services) because agglomeration industries create pure rents which might be taxed (see e.g. Andersson and Forslid (2003), Baldwin and Krugman (2004)).

In the second part of the paper we extend the model to a multi-region economy, i.e. allowing for several regions, of which some have a city whereas others do not. The number of cities is determined endogenously. We find that there will be too few and too small cities as long as there is no interregional mobility. If people are interregionally mobile, the result might be that there will be too many large cities and too many people living in rural regions, whereas the number and size of smaller cities will be too low.



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